U.G. 6th Semester Examination - 2022

PHYSICS

[HONOURS]

Course Code : PHY-H-CC-T-14
(Statistical Mechanics)

Full Marks : 40 Time : $2\frac{1}{2}$ Hours

The figures in the right-hand margin indicate marks.

Candidates are required to give their answers in their own words as far as practicable.

1. Answer any **five** questions: 2×5=10

a) The Fermi energy of free electrons in Silver atoms at 0 K is 5.51 eV. What is the average energy per electron?

b) Show that the relation between partition function Z and average pressure $\overline{p}$ is

$$\overline{p} = k_B T\left(\frac{\partial \ln Z}{\partial V}\right)_T.$$

where the symbols have their usual meaning.

c) Define ensemble average.

d) State the principle of equipartition of energy.

e) Write down Bose Einstein distribution function. What are the basic assumptions used in the derivation of the Bose Einstein distribution function.

f) Find differences among microcanonical, canonical and grand canonical ensembles.

g) What is ergodic hypothesis?

h) What do you mean by extensive parameter and intensive parameter? Give some examples of both.

2. Answer any **two** questions: 5×2=10

a) i) Define μ-space and Γ-space. Can different phase trajectories insect each other?

ii) Deduce classical Liouville equation. 2+3

b) Consider a system of N identical non-interacting linear harmonic oscillator, whose Hamiltonian is given by

$$H\left(p_i, q_i\right) = \frac{p_i^2}{2m} + \frac{1}{2}m\omega^2 q_i^2 \quad \left[i = 1,2,3,........\right].$$

Treating the system as canonical ensemble.

i) Calculate the partition function of the system.

ii) Show that its internal energy is $U=Nk_BT$.

iii) Find an expression for the entropy of this system. 2+1+2

c) Consider an isolated system of four non-interacting spins labeled l, 2, 3 and 4, each with magnetic moment m, interacting with an external magnetic field B. Each spin can be parallel ('up') or antiparallel ('down') to B, with the energy of a spin parallel to B equal to $\varepsilon = -mB$ and the energy of a spin antiparallel to B equal to $\varepsilon = +mB$. Let the total energy of the system be $E = -2mB$.

i) How many microstates of the system correspond to this macrostate? Enumerate these microstates.

ii) What is the probability that the system is in a given microstate in equilibrium?

iii) What is the probability that a given spin points up? Use this probability to compute the mean magnetic moment of a given spin in equilibrium. l+1+(1+2)

d) Consider a System of N>>l weakly interacting particles, each of which can be in quantum states with energies 0, ε, 2ε, 3ε,...

i) If the system is in its ground state, what is its entropy?

ii) If the total energy of system is ε, what is its entropy?

iii) What is the change in entropy of the system if the total energy of the system is increased from ε to 2ε? $1+1\frac{1}{2}+2\frac{1}{2}$

3. Answer any **two** questions: 10×2=20

a) A system of N distinguishable particles are distributed in two non-degenerate levels with energy $+\in$. Calculate the partition function of the system. Hence find out the internal energy and specific heat of the system. For specific heat obtained earlier discuss the two limiting cases i.e. at $T \rightarrow 0$ and $T \rightarrow \infty$ and show it graphically. For this system show that the maximum entropy is Nk_B 1n2.

2+(2+2)+2+2

b) i) Treating the blackbody radiation as a gas of photons, deduce Planck's blackbody radiation formula. Hence plot the variation of energy density as a function of wavelength at three different temperatures. Show that Planck's law reduces to Wien's law for $h\nu >> kBT$ and to Rayleigh-Jeans law for $h\nu << kBT$.

ii) Estimate the surface temperature of the red giant star Aldebaran, given that it emits radiation with maximum intensity at a wavelength of 7250 Å. You can use the fact that the maximum intensity of solar radiation is at wavelength 5000 Å and corresponds to a surface temperature of about 5780K. (4+2+2)+2

c) i) Plot and compare Fermi-Dirac, Bose-Einstein and Maxwell-Boltzmann distribution function as a function of energy.

ii) Show that at $T = 0$, the average energy of an electron in a metal is $\frac{3}{5}E_F$ where, E_F denotes the Fermi energy.

iii) Four weakly interacting particles are confined to a cubical box of volume V with the energy of any one particle of the form

$$E = \frac{\pi^2\hbar^2}{2mV^{2/3}}\left(n_x^2 + n_y^2 + n_z^2\right)$$

where n_x, n_y and n_z are natural numbers. What is the energy of the system at absolute zero if the system is Fermionic? Ignore spin. 3+4+3

d) i) For a 3-dimensional ideal Bose gas, deducing the condensation temperature explain the phenomenon Bose-Einstein condensation.

ii) Show that for a 2-dimensional ideal Bose gas there is no Bose-Einstein condensation.

iii) Seven Bosons are arranged in two compartments. The first compartment has 8 cells and the second compartment has 9 cells of equal size. What is the total number of microstates for the macrostate (3, 4)? (3+2)+2+3

————————